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is more convenient to speak of the variable x than the variable number represented by x. Hence the symbol definition of a variable has a slight advantage with respect to brevity and it makes it easier for those whose desire to avoid the term variable number.

The main object of the preceding remarks is to call attention to the principal elements involved in the different definitions of the term variable, and to give some reasons for the existence of these differences. The teacher is frequently called on to justify his views on points where different usages exist and it is desirable to be able to give references as well as reasons even if the difference appears somewhat trivial. If all numbers were essentially constants I presume most of us would agree with the foot-note quoted from Cesaro to the effect that there is no such thing as an infinite number, for we cannot conceive of any constant number which has such a distinctive character; but it is not so difficult to conceive of a variable which increases without limit and it is convenient to express this concept by saying that the variable is infinite. The ideas of infinite number and variable number are therefore closely related. The existence of the number zero, on the contrary, does not necessarily involve the concept of variability.

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NOTE ON A FORMULA FOR THE SUMMATION OF CERTAIN POWER SERIES.

By HENRY KEMMERLING, Scranton, Pa.

Weierstrass's factor-theorem* is

$$fx = e^{Gx} \prod_{n=1}^{\infty} \left(1 - \frac{x}{a_n}\right) e^{x/a_n + \frac{1}{2}(x/a_n)^2 + [1/(n-1)]\,(x/a_n)^{n-1}},$$

where fx has zeros at the points a_1, a_2, a_3, ...

Taking logarithms and differentiating,

$$\frac{f'x}{fx} = G'x + \sum_{n=1}^{\infty} \left(\frac{1}{x-a_n} + \frac{1}{a_n} + \frac{x}{a_n{}^2} + \dots \frac{x^{n-2}}{a_n{}^{n-1}}\right).$$

Then if $\left(\frac{f'x}{fx} - G'x\right)$ is Px, we have

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*Harkness and Morley's *Introduction to Analytic Functions*, p. 199

$$Px=\sum_{n=1}^{\infty}\left(\frac{1}{x-a_n}+\frac{1}{a_n}+\frac{x}{a_n{}^2}=\ldots\frac{x^{n-2}}{a_n{}^{n-1}}\right)=-\sum_{n=1}^{\infty}\left(\frac{x^{n-1}}{a_n{}^n}+\frac{x^n}{a_n{}^{n+1}}+\ldots\right). \qquad (1)$$

Maclauren's formula is

$$Px=P0+P'0\frac{x}{1}+\ldots\, P^n0\frac{x^n}{n!}+\ldots \qquad (2)$$

Comparing the general terms of (1) and (2), we have

$$P^n0\frac{x^n}{n!}=-\sum_{n=1}^{\infty}\frac{x^n}{a_n{}^{n-1}}, \text{ or } P^n0=-n!\sum_{n=1}^{\infty}\frac{1}{a_n{}^{n+1}}. \qquad (3)$$

As an application of (3) take $fx=x^3-6x^2+11x-6$, whose zeros a_1, a_2, a_3, are 1, 2, 3.
$f'x=3x^2-12x+11$.

$$Px=\frac{f'x}{fx}=\frac{11-12x+3x^2}{-6+11x-6x^2+x^3},$$

which by division,

$$=-\tfrac{11}{6}-\tfrac{49}{36}x-\tfrac{251}{216}x^2-\ldots.$$

Using (3), and reducing, we have,

$$\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}=\tfrac{11}{6},$$

$$\frac{1}{a_1^2}+\frac{1}{a_2^2}+\frac{1}{a_3^2}=\tfrac{49}{36},$$

$$\frac{1}{a_1^3}+\frac{1}{a_2^3}+\frac{1}{a_3^3}=\tfrac{251}{216},$$

. . . .

Again let $fx=1-\sin\frac{\pi x}{2}$, which has *two* zeros at each of the points $+1, -3, +5, -7, \ldots$

$$f'x=-\frac{\pi}{2}\cos\frac{\pi x}{2},$$

$$Px=\frac{f'x}{fx}=\frac{-(\pi/2)\cos(\pi x/2)}{1-\sin(\pi x/2)},$$

$$P'x=\frac{-(\pi^2/4)}{1-\sin(\pi x/2)},$$

$$P''x=\frac{-(\pi^3/8)\cos(\pi x/2)}{[1-\sin(\pi x/2)]^2},$$

$$P'''x=\frac{(\pi^4/16)\sin(\pi x/2)[1-\sin(\pi x/2)]-(\pi^4/8)\cos^2(\pi x/2)}{[1-\sin(\pi x/2)]^3}.$$

$P0=-\pi/2,\quad P'0=-\pi^2/4,\quad P''0=-\pi^3/8,\quad P'''0=-\pi^4/8.$

Applying (3),

$$-\frac{\pi}{2}=-2\left(\frac{1}{1}+\frac{1}{-3}+\frac{1}{5}+\frac{1}{-7}+\ldots.\right)$$

$$-\frac{\pi^2}{4}=-2\left(\frac{1}{1^2}+\frac{1}{(-3)^2}+\frac{1}{5^2}+\frac{1}{(-7)^2}+\ldots.\right)$$

$$-\frac{\pi^3}{8}=-4\left(\frac{1}{1^3}+\frac{1}{(-3)^3}+\frac{1}{5^3}+\frac{1}{(-7)^3}+\ldots.\right)$$

$$-\frac{\pi^4}{8}=-12\left(\frac{1}{1^4}+\frac{1}{(-3)^4}+\frac{1}{5^4}+\frac{1}{(-7)^4}+\ldots.\right)$$

Reducing above we get the familiar series,

$$\frac{\pi}{4}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots.$$

$$\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots.$$

$$\frac{\pi^3}{32}=\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\ldots.$$

$$\frac{\pi^4}{96}=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\frac{1}{7^4}+\ldots.$$

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